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MECHANISM AND CAUSALITY IN PHYSICS[1]

THE fundamental questions of physics which from the days of Aristotle to those of Descartes and Kant supplied so much of the stimulus and substance of philosophical reflection, are now receiving scant attention from professional philosophers. We are now, indeed, witnessing the rapid passing of the old conception of philosophy as the official critic of the fundamental principles and presuppositions of the special sciences. As the special sciences have developed and have become more and more technical, philosophers have become more modest and grown content to cultivate a special field of their own, sharply distinguished from that which is the proper domain of any other academic department. Now, while modesty is undoubtedly a precious virtue, it is also frequently an easy excuse for evading a difficult task; and while the meticulous delineation of fields may be a necessary postulate of academic life, it may, perhaps, not always be the most effective method of gaining philosophic insight. It is certainly futile to appeal, as is the manner nowadays, to the "method" of science unless we take the trouble to become familiar with what are actually the methods of the sciences; and it is hazardous to accept the "results" of the sciences unless we know how much unconscious, but none the less antiquated, metaphysics has entered into their make-up. It is a distinguished physicist who has lately reminded us that a metaphysics is no sounder because it is held unconsciously or professed by one who is not professionally responsible for it.[2]

As the term mechanics has been freely used and abused, a few distinctions at the outset may clarify the discussion. In the first place, we must distinguish between the mechanical and the physical. The term mechanics as used by physicists[3] denotes that branch of

[1] Read before the American Philosophical Association, December, 1911. This study forms part of a book on *The Principles of Natural Science*, whose publication has been unavoidably delayed.

[2] Maclaurin, *The Theory of Light*, p. 7.

[3] Continental usage has been fixed in this respect since Varignon's *Nouvelle Mécanique* (1667). In England the term mechanics is sometimes restricted to the study of machines, but Thomson and Tait (*Elements of Natural*

physics which studies the motions of masses (considering equilibrium as a special case or limit of motion). Now there are physical phenomena such as light, magnetism, *etc.*, which are not *prima facie* phenomena of motion, and no physicist claims that all these have, as a matter of fact, been satisfactorily explained on mechanical principles.[4] It may seem altogether superfluous to point out that the belief that with increasing knowledge physics may be completely reduced to mechanics, is a pious hope, that had better be explicitly stated rather than be covertly implied in the use of a term. Yet, failure to keep the distinction between the physical and mechanical clearly in mind has actually caused a great deal of confusion in the discussion of the issue between mechanism and vitalism.[5]

It seems also necessary to distinguish between mechanism and determinism. The changes of a physical system may be treated as a function of a number of variables, the mechanical conditions of the system as expressed in geometric coordinates being only one set of these variables.[6] It follows, therefore, that a system may be determined in its mechanical features and physically undetermined, without any breaks or discontinuity in our laws of nature. Moreover, the events in a Kingdom of Heaven or the inner life of a Leibnitzian monadology might be absolutely determined and yet not be, except in an obviously metaphorical sense, mechanical.

A third obvious distinction, which has actually been ignored to the detriment of clear thinking, is that between mechanical phenomena and phenomena expressible in certain kinds of differential equations. It has been widely supposed that, whenever the laws of any branch of

Philosophy, art. 1) are not justified in claiming the authority of Newton for this—witness the introductory paragraph of his *Principia.* Besides, it is well to remember that when there were no steam or electric engines ''rational'' mechanics could only deal with *vis viva* or masses in motion.

[4] Boltzmann and Planck, the most distinguished physicists to defend the mechanical methods of the classical physics, have pointed this out clearly—see *Wiedemann's Annalen,* Vol. 57 (1896), pp. 64, 65, and Planck's *Acht Vorlesungen über theoret. Physik,* p. 64. Boltzmann says explicitly: ''The possibility of a mechanical explanation of the whole of nature has not been demonstrated, yea it is hardly probable that we shall completely reach that goal.'' (*Op. cit.,* p. 70.)

[5] Even as careful a thinker as Professor Lovejoy (*Science,* April, 1911, p. 612) fails to note that mechanism and vitalism are not exclusive alternatives, and that a physico-chemical explanation of biologic facts is not necessarily a mechanical one. Loeb, the leader of those who call themselves mechanists, is as far as Driesch from believing that the phenomena of life can be explained by the motion of particles. In the light of recent progress in physical chemistry, also, it is hazardous to assert the existence of a greater gap or discontinuity between physics and chemistry than between mechanics and other branches of physics such as optics, theory of magnetism, or even the theory of elasticity.

[6] J. J. Thomson, *The Application of Dynamics to Physics.*

physics, *e. g.*, those of electricity, can be expressed in the Lagrangian form, something has been achieved in the way of a mechanical explanation.[7] It is interesting to note that Maxwell, whose procedure in his great treatise on *Electricity and Magnetism*[8] is largely responsible for this impression, had previously been careful to point out that the mathematical form of the relation between different quantities might be the same though their physical natures were different.[9] But mathematical analogies have always proved such a fruitful source of physical discoveries that physicists have been too prone to lose sight of the fact that mathematical analogy does not mean physical identity. This confusion has also been furthered by the ready way in which people confuse logical with historical priority. Thus, it has actually been argued[10] that since Lagrange's equations were first derived from mechanical considerations, they are not likely to be general forms of natural law, and hence everything expressed by them must be ultimately mechanical. The date of derivation is, however, no part of the mathematical or physical meaning of these equations. Like other equations, they state the mutual implication of certain functions of variables, and the physical meaning of these equations depends upon the interpretation or meaning that we attach to the independent variables. Clearly, the general form (and even the method of derivation) of the Lagrangian equations does not demand that their variables should be masses and velocities rather than electric charges and their intensities. All sorts of different phenomena, social, economic, or physical, as well as electrical or thermal, may have their variations expressed by the same equations, precisely as they are subject to the same laws of the multiplication table.[11] The fact, therefore, that the laws of electricity can be made to assume the same form as the laws of mechanics no more proves the primacy of the mechanical than it proves the primacy of the electrical.

I

1. It is one of the unfortunate results of Ward's *Naturalism and Agnosticism* that it has strengthened the unhistorical notion that mechanism, *i. e.*, the mechanical interpretation of nature, is inconsistent with ontologic idealism. While it is true that mechanism has frequently been developed in the interests of physical monism or

[7] Larmor, *Æther and Matter*, p. 83. Maxwell, *Electricity and Magnetism*, p. vii. Combebiac, *Les Actions à Distance*, appendix.

[8] Part IV., Ch. 6–7.

[9] *Scientific Papers*, II., p. 218.

[10] Combebiac, *op. cit.*, p. 81.

[11] Petrovitch, *La Mécanique des Phénomènes fondée sur les Analogies*, esp. pp. 7–20. As an illustration from the realm of economics see the *Comptes Rendues de l'Académie des Sciences*, 1911, p. 1129.

materialism, it must not be forgotten that the mechanical view of nature was fashioned by the founders of modern idealism, Descartes, Spinoza, Liebnitz, and Kant; and to-day, it is idealists of such diverse schools as Wundt, and Fullerton, who contend that the mechanical point of view is necessary for physical science.

It is precisely this supposed necessity that is in need of critical examination. Why must all physical phenomena be viewed as ultimately so many different forms of motion? It is to be observed that the classic science of mechanics is a deductive system of propositions, all deducible from Newton's Three Laws of Motion or D'Alembert's Principle in its Lagrangian or Hamiltonian form.[12] But an examination of Newton's laws and D'Alembert's principle, or the principle of least action in its Hamiltonian form, does not reveal any of them to possess inherent logical necessity; nor has any valid *a priori* reason ever been adduced why all events in nature should be deducible from these laws. The attempts of philosophers like Descartes, Kant, or Wundt, or even of physicists like D'Alembert or Playfair, to prove these laws, hardly need any refutation.[13] Careful examination of them readily shows that they either move in a circle, taking for granted the very principles which they pretend to prove, or else they appeal to principles which are no more self-evident (whatever that may mean) than those they wish to prove. But it is not necessary to examine these *a priori* proofs, since we are in possession of experimental facts tending to show that these principles are not at all universally true, but are only first approximations, *i. e.*, true only within certain limits. Thus, the Newtonian assumptions of the constancy of mass and the proportionality between force and acceleration are now regarded as true only of tangible masses at ordinary velocities (ranging up to the paltry 18 miles per second with which the earth moves in its orbit). When we come to the small particles which compose the cathode rays or the β rays of radium, moving with velocities comparable to that of light, recent experimental physics has been forced to assume that the masses no longer remain constant but vary with the velocity. Thus, even apart from the Einstein-Minkowski relativity theory—the only one that explains the Michaelson and Morley experiments—there is evidence for imposing a superior limit on possible velocities. Hence, the principle of the composition of velocities, or that acceleration varies directly as the force, is no longer of universal application. At any rate, there can

[12] In *Crelle's Journal*, Vol. IV. (1829), p. 233, Gauss has a demonstration that no other principle will ever be necessary.

[13] Descartes, *Principia*, II., art. 23. Kant, *Met. Anfangsgründe*, Pt. III. Wundt, *Prinzipien der mechanischen Naturlehre*. D'Alembert, *Dynamique*, pp. 7, 64. Playfair, *Outlines of Nat. Philosophy*, p. 26.

be little doubt that the question, What fundamental principles of mechanics are actually true? can not be determined *a priori* but only by examining the experimental evidence—which involves elements of contingency.

Similar considerations apply to the attempts to prove *a priori* that all physical phenomena must ultimately be mechanical, *i. e.*, consist of the motions of material particles. As such arguments have only recently been repeated by Wundt and Meyerson, it may be well to examine them here.

The gist of Wundt's argument is that it contradicts our perception to assert that an object can change and still remain the same,—except in the case of spatial change.[14] With all due respect, I must urge that this is sheer dogmatism. Our perceptions certainly do not contradict the assertion that an object can be now hot and subsequently cold, or that the same piece of soft iron can be at one time magnetic and subsequently not so—certainly there is no more contradiction here than in saying that the same object can be now in one place and now in another. The contradiction in saying that a house can remain the same though the color of its roof has been changed, is a contradiction which exists not in perception but only in a conceptual system which arbitrarily defines the identity of an object to consist in the maintenance unchanged of all its possible properties except its spatial coordinates. If an object can change its location and still retain its identity, why may it not similarly change its color, its thermal or electric properties? The assumption that the only possible changes of reality are spatial is simply the mechanical dogma over again in a different guise, and we have here no genuine proof but a *petitio principi.*

The same logical fallacy of supposing that facts of qualitative change are ruled out from reality because they contradict an arbitrary definition of identity, underlies the remarkably learned and charmingly written book of Meyerson, *Identité et Réalité.*[15]

Remembering, however, that good causes are frequently defended by bad arguments, we ought to be on our guard as to whether we can not find a better reason for the belief in the primacy of spatial change, a belief which has persisted since the foundation of modern physics. Such a reason, I believe, is to be found in the historic fact that only by reducing physical changes to phenomena of motion was it possible for the men of the Renaissance to overthrow the scholastic physics of illimitable occult qualities and to build up instead a quan-

[14] *Prinzipien der mechanischen Naturlehre,* pp. 179 ff. In substance the same argument is repeated in all his other works. *Cf. Logik,* II., p. 225, ff.; *System,* p. 423.

[15] Pp. 98–99. A similar argument was adduced by that lonely thinker, Spir, *Denken und Wirklichkeit,* p. 424.

titative physics capable of fruitful mathematical development. This was reinforced in the minds of men like Kepler and Galileo by the Neo-Platonic doctrine that the body of nature was composed in purely geometric terms. It is under the influence of the latter that Galileo brought forth, in his *Il Saggiatore,* the modern doctrine of the distinction between primary and secondary qualities. If only extension and motion are truly existent in nature, and colors, tastes, temperatures, *etc.,* are mere names or subjective products, then a true physics can be had only by reducing all phenomena to those of motion. The remarkable rapidity with which this doctrine was at once adopted from Galileo by men like Kepler, Descartes, and Hobbes, shows what a fundamental need of the time it met. Nevertheless, it is to be noted that the only fairly consistent attempt to banish all qualities from physics, *viz.,* the Cartesian attempted reduction of physics to geometry, broke down under the criticism of Gassendi, Newton, and Leibnitz. Atomists, Leibnitzians, and Newtonians, in turn, postulated besides space and matter, primitive qualities, forces, and the properties of repulsion and attraction, respectively. Moreover, as our instruments of measurements have increased, and as our mathematical methods have developed, changes in all sorts of qualities, such as illumination, elasticity, or electric charge, have become just as capable of mathematical development as changes of distance. Hence, the motive for reducing everything to spatial properties is no longer a living one. Doubtless scientific physics always endeavors for technical and esthetic reasons to reduce the number of fundamental qualities to a minimum consistent with the known diversity of facts. But this is distinct from the pretended *a priori* proof that all changes must ultimately turn out to be spatial. Against all the latter attempts it is significant to call attention to recent experimental work which tends to show that mass phenomena are of electric origin and that, therefore, electricity may turn out to be more fundamental than mechanics.[16]

2. There are, however, philosophers who distrust dialectic *a priori* arguments and even reject the distinction between primary and secondary qualities, who yet believe, as does Professor Fullerton,[17] that all that takes place in the world must be explicable according to mechanical laws. Professor Fullerton frankly admits that the world is not known to be such a system, but the vision of it, he says, is re-

[16] See the last chapter of Righi's *Modern Physical Theory;* J. J. Thomson, *The Electrical Nature of Matter.* For the earlier statement of the theory, see Larmor in the *Transactions of the Royal Society,* 186 (1895), p. 617 and *Wien, Archives Neerl.,* 1900, p. 96. Kaufmann's experiments are reported in the *Götting. Nachrichten,* 1901, p. 143.

[17] *System of Metaphysics,* pp. 147, 226.

vealed to the eye of faith.[18] This faith, I suppose, is based on a popular impression that the mechanical view has been making steady progress towards a complete explanation of the physical universe, and that it is, therefore, reasonable to hope that the hitherto unconquered fields will in the course of time yield to the sway of mechanical explanation. This is, however, a view which finds no support in any actual history of physics. Indeed, the most competent historian of physical science arrives at the very opposite conclusion.[19] Even if we do not share Duhem's view as to the final bankruptcy of the mechanical view, there can be no doubt that to the conscientious reader of the history of physics there is no such continuous progress towards a mechanical millennium as is pictured in the popular myth. It is easy to show that throughout the history of physics there have never been wanting fruitful researches carried on in utter independence of the mechanical hypothesis: the foundation of thermo-dynamics by Fourier, of electro-dynamics by Ampère, and the phase rule by Gibbs, are striking and well-known instances. The history of mechanics also shows a perpetual see-saw between those who are partisans of the conflicting claims of motion, the atom, or force, as the primary and all-sufficient category. Thus, the purely kinematic view of mechanics, with its æther and vortices, which seemed to have died with Descartes, was revived by the vortex-ring hypothesis of Helmholtz and Kelvin, by Larmor and others in their attempts to derive matter from æther, and in a different guise by Hertz in his brilliant but uninfluential *Mechanics*. The atomic hypothesis, brought into modern physics by Gassendi, Huygens, and Boyle, was eclipsed by the physics of forces of Newton and Leibnitz (united in Boscovich), and was revived again by Dalton and Avogadro in the early part of the nineteenth century. It suffered some eclipse in the latter part of the nineteenth century—witness Berthelot, St. Claire Deville, and Ostwald—and is now to the forefront again in the form of the electron theory.[20] Nor has the Newtonian dynamics had an unchecked

18 *Op. cit.*, p. 227.

19 Duhem, *L'Evolution de la Mécanique* (1905). See also his *La Théorie Physique* (1906); his *Essai sur la Notion de Théorie Physique* (1905); *Le Mixte* (1904); and *Introduction à la Mécanique Chimique* (1903).

20 It is of course only analogically that the present electron theory may be called atomic. In one sense, however, it is an emphatic refutation of the old conception of the *atom* as absolutely *indivisible*. The basis of the present electron theory is not any *a priori* or philosophic necessity, but the empirical discovery that many physical facts involve multiples of a certain amount of electricity. As to what physical fact corresponds to this mathematical unit, it would be hazardous to assert with any assurance in the present state of our knowledge. I may, however, add that the phenomenalistic view that the physical atom is a mere symbol or mental figment ignores the vast mass of empirical evidence which makes the existence of atoms (*i. e.*, *physical* indivisibles) as

career. Its great triumph in astronomy made its immediate ascendency irresistible, and for over a century and a half all physical phenomena were viewed as those of miniature astronomical systems, governed by central forces. La Place's treatment of capillarity in his *Mécanique Céleste* is perhaps the most characteristic product of this attitude, in which the attractive and repulsive forces of non-extended points in empty space were regarded as the key to all the secrets of nature. Yet the opposition to the Newtonian concept of gravity as a property of matter—witness the works of Euler and Bernouilli—never completely died out. When La Place confidently announced the permanent completion of the Newtonian system by his explanation of the double refraction of light, a large part of that structure had already been undermined by the labor of Young, Fresnel, and Faraday, which brought back the æther and contact forces and banished action-at-a-distance. But the multiplicity and complexity of the various models of the æther—elastic, labile, solid, fluid, irrotational, gyrostatic, adynamic, *etc.*—soon made physicists weary and brought about a reaction, so that good physicists now prefer to go back to something like an emission theory of light rather than lose themselves in interminable seas of hypothetical mechanisms, besides which the Ptolemaic cycles and epicycles were simplicity itself.[21]

3. A third type of argument, the psychological, is represented by Abel Rey's recent book, *L'Energetique et le Mécanisme.* The substance of M. Rey's contention is as follows: There can be no thought without images, and mechanics is best suited to provide images or models of physical phenomena. Energism or mathematical physics may formulate the knowledge we have, but it can not serve as an instrument of research. The laboratory physicist must work with the mechanical hypothesis in mind. This argument can be supported by many quotations from Lord Kelvin and other British physicists to the effect that to understand physical phenomena means to be able to form mechanical models of them. This, however, is not a statement of a universal law. It is true only of a certain type of mind, of

probable as the existence of King David, Crœsus, or the man Shakespeare. It is only in imagination that we can go on dividing matter indefinitely without changing its specific qualities. This is the case because the imaginary process of division soon gets to a point where the imaginary division is only a duplication of the small magnitude supposed to be divided. In physics, however, we find a preponderance of evidence to indicate that matter is not indefinitely divisible but that there is a limit to this process below which the breaking up of matter, *e. g.*, water, or wood, results in radical changes in its specific properties.

[21] Campbell, *Philosophical Magazine,* 19 (1910), p. 181. Trowbridge, *Am. Journal of Science,* 31 (1911), p. 51.

those who, when they calculate the forces between the heavenly bodies, "feel their own muscles straining with the effort." But as far back as 1870 Maxwell,[22] the most illustrious representative of this type of mind in physics, had recognized the existence also of the abstract and mathematical type, and that "the tenuity and paleness of symbolic expression" had equal rights in science with "the robust and vivid coloring of physical illustration." There is not a single diagram in Lagrange's *Mécanique Analytique,* and a careful reading of it shows that Lagrange had few physical images before his mind as he wrote it. If there are minds that can dispense with diagrams in geometry and mechanics, why not minds that can dispense with mechanical models of physical phenomena? Mechanical models certainly have not as much relevance to physical inquiry as diagrams in geometry, since it can be shown, as Poincaré[23] has done, that whenever a mechanical model is invented to explain physical phenomena, an infinity of other models is possible.

Nor is it true, as a matter of fact, that the mathematical type of mind is impotent to produce great physical discoveries. From the discovery of the laws of planetary motion by Copernicus and Kepler, or of universal gravitation by Newton, to the discovery of the laws of thermal and electric conduction by Fourier and Ohm, or the pressure of light by Maxwell, a long list of most impressive physical discoveries by purely mathematical methods can be drawn. Physicists, like others, are not always the best judges of what is going on in their own minds when they are working, and many who speak a current language of mechanism really carry on their researches by mathematical methods. Did not Maxwell himself arrive at the electro-magnetic character of light by the purely mathematical analysis of the dimensions of the ratio between the electrostatic and electromagnetic unit?[24] The same is true of many of Lord Kelvin's discoveries in thermodynamics.

I pass over M. Rey's argument for mechanism based on the ground that knowledge must proceed from the simple to the complex. Surely the various strains and stresses in the æther or the lateral vibrations in polarized light, are not psychologically simpler than the phenomena of light which the mechanistic hypothesis attempts to explain.

II

All the arguments for the mechanical dogma thus turn out to be vain. But our analysis suggests that *a priori* arguments *against*

22 *Scientific Papers,* II., 220.

23 *Electricité et Optique* (1890), preface.

24 *Scientific Papers,* I., pp. 577 ff; II., pp. 137 ff; and *Electricity and Magnetism,* IV., Ch. XIX.

mechanism would similarly prove ineffective. The present actual decline of mechanical explanation in physics may render the full revival of such explanations unlikely but not impossible.

It is a curious and noteworthy fact—worthy of greater attention than it has yet received from those interested in the drama of human thought—that philosophic criticism of physical procedure has almost always gone entirely unheeded. Apparently valid arguments by men like Stallo and Ward to the effect that the mechanical hypothesis was inconsistent with itself and inconsistent with the facts, have failed to exert any noticeable influence on physics.[25] The fact seems to me to be that neither of these inconsistencies is of great moment to the physicist. A final and finished account of the physical universe must doubtless be free of contradiction, but the physicist bent on exploring the facts may well use two contradictory hypotheses—such as the continuous and the discrete nature of matter—for the purpose. To the physicist, it must be remembered, an hypothesis is not an impeccable account of what he already knows but an anticipation of experience to guide his search; and while there can be no search at all without some hypothesis to point to the object sought for, it is not at all necessary that the anticipation should be absolutely complete and accurate. A large element of vagueness and indetermination in our hypotheses is not at all incompatible with its suggestive quality. Indeed, it may even be helpful in this regard in keeping the mind open to a greater number of possibilities.

Nor is a contradiction between a theory and the facts necessarily a fatal objection. Physical theories are flexible. If the facts of

[25] Thus, Stallo, and Ward after him, have argued that, on the kinetic theory, atoms must rebound when they clash or else *vis viva* be lost and the laws of mechanics no longer prevail; but if they do rebound, elasticity becomes a fundamental property of matter and the atomic theory no longer offers any explanation of it. (*Cf.*, Kroman, *Unsere Natur Erkenntniss*, p. 315.) Neither horn of this dilemma, however, can be considered fatal. If the atomic theory does not explain elasticity, there are many other facts like the diffusion of gases which it does explain. On the other hand, recent physics has taught us that it is not necessary that the laws applying to ponderable masses should apply in the same way to molecules or atoms. We must guard against the naïve assumption that the laws observed to hold within the limits of the pressures, temperatures, masses, *etc.*, actually observed, must necessarily hold below or above these limits. Maxwell shocked his contemporaries, even the agnostic Huxley, by asserting that the law "two portions of matter can not occupy the same space" has no application to molecules (*Scientific Papers*, II., 33). Yet it is clear that the law in question is not *a priori* necessary but founded on the simple empirical observation that ordinary solid matter has the property of impenetrability. If we had been as familiar with the diffusion of gases, or even with the interpenetration of water and alcohol, the dogma of impenetrability would never have acquired its vogue.

radiation do not fit in with the law of the conservation of energy, an æther can be invented and endowed with just those properties which will make the law true. If, therefore, physicists of a certain type of mind find that illustrative models based on a mechanical hypothesis help them to visualize their problems, it is as vain to argue against them as to argue against their religion or political affiliations. The effective thing in the long run is always the elaboration of the possibilities of some alternative method of explaining all of the facts with less hypothetical elements.

Now, the fundamental postulate of mechanism, as we saw, is the assumption that there is an ultimate structure of things which it is the primary business of the physicist to discover, and even where it has not yet been discovered, he must still be sure that it consists in nothing but the hidden motions of particles. An alternative to this realistic monism of motion has, as a matter of fact, always existed in physics since the days of Ptolemy[26] and Archimedes. But it has only recently been able to obtain sufficient philosophic backing to make it self-conscious and respectable. Since the days of Kant and Comte, physicists need not be ashamed of admitting that their science is not a means of piercing the veil of phenomena and grasping the ultimate reality behind it, but only a method of extending and organizing our knowledge of these phenomena; and the recent revival of pluralism supports those who refuse to believe that all possible changes of the physical universe must be reducible to just one kind of change: namely, motion. It is interesting to note that Comte's views in this matter were determined by Fourier, whose preliminary chapter in his *Théorie Analytique de Chaleur* contains the essence of the matter. Expressions of it may be found in the writings of the founders of mechanics itself. Thus, Galileo states explicitly,[27] "It does not appear to me at present worth while to investigate the causes of natural motion, concerning which there are as many different opinions as there are different philosophers. Some refer them to an attraction towards the center, others assign them to repulsion between the small particles of a body, while still others would introduce a certain stress in the surrounding medium which closes in behind the falling body and drives it from one of its positions to another. But it is not worth while examining all these fantasies. All that is needful is to investigate the properties of accelerated motion and define it in such a way that the momentum of the body increases uniformly in simple proportionality to the time." The

[26] Duhem, *Essai sur la Notion de Théorie Physique* (1908). See also Delambre article on Kepler, in Michaud's *Biographie Universelle*.

[27] *Discorsi e dimonstrazione intorno a due nuove scienze. Opere* (1811), VIII., p. 256.

same attitude was expressed by Newton in his famous adage: "*Hypothesis non fingo.*" By hypothesis, we must remember, Newton meant an explanation not directly derived from phenomena. Thus he says in the concluding *scholium* to the *Principia:* "Hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and gravitation were discovered. And to us it is enough that gravity does really exist, and acts according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies and of the sea."

An even more explicit statement of this we have in Rankine's paper on the *Science of Energetics* (1855). Rankine clearly distinguishes two methods of constructing a physical theory, which he calls the hypothetical and the abstractive. The hypothetical method consists in starting with some hypothesis about something which is not the object of direct perception, and deducing from this supposed constitution the empirical properties. All mechanical theories of physics, *e. g.,* the kinetic theory of gases, illustrate this method. The abstractive method, on the other hand, is described as follows: "Instead of supposing the various classes of physical phenomena to be constituted in an occult way of modifications of motion and force, let us distinguish the common properties which these classes possess and define more extensive classes denoted by suitable terms. For axioms let us frame propositions containing as particular cases the laws of the particular classes of phenomena comprehended under the more extensive classes. So shall we arrive at a body of principles applicable to physical phenomena in general and which, being framed by induction from facts alone, will be free from the uncertainty which must always attach even to those mechanical hypotheses whose consequences are most fully confirmed by experiment."[28] It is to be observed that while mechanical theories of physics are illustrations of the hypothetical method, mechanics, as a branch of physics studying the laws of motion, is itself an illustration of the abstractive method.

Though Rankine was one of the founders of modern thermodynamics and the author of classical treatises on the steam engine and ship building, this paper received very little attention. It came in the heyday of mechanical models, when every one was trying to derive the principles of energy from the principles of mechanics. These efforts, however, soon came to a standstill. The kinetic theory

[28] Rankine, *Miscellaneous Papers,* p. 245.

of gases struck a rock in the problem of the equipartition of energy, being unable to harmonize the theory with the behavior even of diatomic gases.[29] More particularly it was soon realized that the principle of entropy or degradation of energy—the general fact that physical phenomena are in a given direction and irreversible—could not be explained on mechanical principles. (Thus, two gases will diffuse themselves one in the other, but will not conversely separate themselves spontaneously.) Maxwell, Boltzmann and Gibbs realized this, and introduced the notion of a statistical as opposed to a mechanical knowledge of physical phenomena. Imagine an almost infinite number of particles moving at random with various velocities, and one can compute on the basis of the various degrees of freedom and the principles of statistical probability what the total effect will be. The famous example of the sorting demon was introduced by Maxwell to show that the second law of thermodynamics was not mechanically necessary but had only statistical certainty. This has recently been reinforced by M. Gouy's investigations on the Brownian movements, which indicate that what we ordinarily call thermal equilibrium, *i. e.*, stable, uniform distribution of temperature, is only statistically so when we consider sensible volumes, but does not hold within microscopic volumes, so that the second law of thermodynamics is not applicable within them.

As the laws of thermodynamics are empirically verifiable and independent of all atomic or other hypotheses as to the ultimate structure of things, the enormous success which followed the introduction of this method into physical chemistry by Gibbs, Van der Waals and Van't Hoff, gave support to the empirical or descriptive theory of physical science upheld by Kirchoff, Avenarius, Mach and Duhem.

Before examining the philosophic significance of this theory, it is well to note what it has actually meant for physics. No one can compare the prevailing tone of physical theory to-day with that of a generation ago without noticing the greater recognition to-day of the provisional, empirical, pluralistic, and yet thoroughly mathematical character of physics. No one asserts nowadays, as did Maxwell, that atoms never change and are to-day as fresh as when they came out of the hands of the Creator. Formerly we used to be told that when hydrogen and oxygen combine to form water, the two substances, as represented by the H and O atoms, remain the same, though most of the properties of the H_2O molecule in no wise follow from those of the H and O. Now reputable authorities on physical chemistry, like Ostwald and Duhem,[30] find it more serviceable to re-

[29] See the various papers of Rayleigh and the preface to Gibbs' *Statistical Mechanics.*

[30] Duhem, *Le Mixte,* p. 165. Ostwald, *Lehrbuch d. Allgem. Chemie,* II., pp. 5–9.

turn to the Aristotelian conception of change and to suppose that when an electric spark causes the H and O to combine, the H and O both disappear and a third something, namely, water, takes their place. Modern physics has learned to be suspicious of eternal substantial forms, and is not awed by the scholastic dogma *ex nihil nihil fit, nihil in nihilo.* We no longer think that because the white light that enters a prism issues in the form of many colors, it follows that the white light actually contained all the various colors[31] and we are careful not to say that when a cool body is brought into the presence of a warmer body, the heat gained by one is precisely or identically that which is lost by the other. But perhaps the most striking illustration of the point I wish to make is to be found in the current statement of the law of gravity, which has so long served as the typical law of nature. Instead of asserting dogmatically that every particle of matter attracts every other particle precisely as the product of the masses and inversely as the square of the distance, careful physicists, like Poynting and Thomson, point out that astronomic observation is by no means decisive on this point and that all we can say is that when we take large masses like the planets, the mean results fit in with our formula.[32]

From this point of view the classic notion of absolutely uniform causation, *i. e.*, laws of nature holding with absolute accuracy for the smallest atom as well as for the largest star cluster, can be replaced by the more modest doctrine of statistical averages. Our knowledge of physical phenomena is like that of social phenomena when studied through such facts as marriage rates, birth rates, tables of exports and imports, *etc.* The great difference between physical and social phenomena would thus be due to the fact that in the latter, individual variations obtrude themselves, while in the physical realm the constituent individuals or atoms are for the most part beyond our range of observation.

There are philosophers to whom the slightest suggestion of contingency in the physical world or any doubt as to whether everything does happen absolutely in accordance with universal laws, is an atrocious and unpardonable blasphemy. But whatever may be said for the sublime faith back of this attitude, it surely is not necessitated by the experience of the physicist who works with instruments of precise measurement. Laboratory workers know how difficult it is to get phenomena to repeat themselves even approximately, *i. e.*, within the range that we call the limit of probable error, and they will readily subscribe to the statement in Chwolson's

31 Wood's *Physical Optics,* Ch. XXI.

32 Poynting and Thomson, *Properties of Matter,* p. 46.

great international text-book, that when we study physical phenomena more closely we can convince ourselves that there is almost no physical law which can be exactly verified.[33]

I do not want on this occasion to press the hypothesis of our great American thinker, C. S. Peirce, that there is a domain of radical indeterminism, that besides the variations due to errors of observation, there are variations due to the fact that our physical laws do not express with absolute accuracy the actual behavior of things. But modern physics is beginning to recognize more and more the point made by Poincaré[33a] that the simplicity of Newtonian laws may be the result of averaging large numbers of very complicated phenomena, in accordance with the well-known fact that the larger the number of cases considered, the simpler the expression of the prevailing type. Spectrum analysis and other evidence as to the structure of matter suggest that an atom of sodium may have a structure as complicated as that of a piano or stove, and the variation in the behavior of the atom may consequently be as great as that of these somewhat capricious objects. But when we remember that the number of atoms in a pin-head is greater than that of all the human beings now alive, we can readily understand why any tangible piece of sodium behaves so like any other piece.

The principle of uniformity of nature is usually stated thus: like causes produce like results. But in physics, as in social science, we never have the entire identical situation repeating itself. What we observe is that when the antecedent situations are alike, the sequences are also alike. Now likeness is a matter of degree, *i. e.*, it depends on the fineness of our classification. When we say water freezes at 32° F. we regard all samples of water as alike, and the result is approximate enough when measured by an ordinary thermometer. We may, however, treat our water as consisting of different samples having, *e. g.*, different degrees of density, and notice slight variations in the reading—provided our thermometer is adequately graded. The causal relation is thus simply a statistical correlation between the heights of columns of mercury and the freezing points of the various samples of water.

III

The considerations involved in the last section are so well established in the daily procedure of scientific investigation, that any form of rationalism which puts its face against them is bound to come to

[33] *Traité de Physique*, I., p. 29. *Cf.* Poincaré, *Value of Science*, Pt. III., art. 4–5, and Thomson and Tait, *Natural Philosophy*, I., Ch. III.

[33a] *Thermodynamique*, p. vii.

grief. The old rationalistic conception that the principles of mechanics are *a priori* self-evident axioms which will never be successfully attacked,[34] has been effectively disposed of by the rise of non-Newtonian mechanics. The primary laws of mechanics, as of any other branch of physics, are now seen to be logically contingent, *i. e.,* they are not to be derived from the non-temporal laws of logic.[35] Their contraries are possible hypotheses. There is nothing illogical or inconsistent in supposing the Newtonian laws of motion or the formula of gravitation to be grossly inaccurate. Not a single established fact of physics but its absolute accuracy has now been rudely shaken by recent experimental work in Brownian movements, radio activity, the phenomena of radiant energy, *etc.* But while all this fortifies the view that natural laws are contingent and only statistically true, it seems to me an unseemly intellectual haste to jump to the conclusion of the positivism or phenomenalism of Mach, Pearson and their followers, who assert that the world simply is, and that all necessary relations are fictions or mental products.

We may grant at the outset that the positivists are right in regarding the popular use of the *word* cause as embodying remnants of primitive animism. When we popularly speak of things causing something else, we undoubtedly tend to attribute to things something analogous to human compulsion, something of muscular tension or the feelings of activity and passivity when we wilfully push or are pulled contrary to our will. Such animism is out of place in modern scientific physics. The Humian analysis of causation and its replacing of the ideas of production, of power and force (as synonyms of compulsion) by the idea of regular sequence, was the *coup*

[34] This used to be the basis for preferring physical deductions from the laws of mechanics rather than from the laws of thermodynamics. (J. J. Thomson, *op. cit.*)

[35] Similar considerations hold true of the mixture of logic and psychology which passes as epistemology. To derive the fundamental laws of physics from the laws according to which the mind operates, does not really remove this contingency. There is no reason to suppose that the laws according to which the mind works are absolute constants, and the only evidence we have as to the way in which the mind operates at its best is the changing body of actual science. There is a close and suggestive parallel between the attitude of modern epistemology to physics and that of the older theology. The older theology tried to derive the truth of physics from the will of God. Neo-Kantian epistemology tries to derive it from the ways in which the mind operates. The physicist may believe as much as he pleases in "the ways of the mind" as he does in the "will" of God. But he must not introduce them as principles of physical explanation, for the simple reason that they are not principles of determination. We have no scientific way of telling the way the mind works or the will of God, except by examining the results.

de grâce which modern thought administered to the scholastic physics of occult qualities and powers. If we do sometimes find authoritative physicists still speaking of the operation of forces in an anthropomorphic way, or lapsing into the popular manner of speaking of heat or gravity as causes, we must remember the great difficulty of freeing ourselves completely from prevailing popular use of words, and the even greater difficulty of expressing ourselves vividly without the use of metaphors, of which anthropomorphism supplies the bulk. Technical and mathematical language, however, is surely, if slowly, replacing expressions of causal relations with mathematical functions or equations, which are neutral to all anthropomorphic hypotheses. In formulating Newtonian laws of motion in popular language, physicists may still use such phrases as: bodies acted on by forces, *etc.* But when the physicists' actual deduction from these laws is carefully examined, we find actions replaced by changes in certain physical coordinates or parameters, and that "force" denotes simply mA, the product of the number of units of mass by the acceleration or rate at which the velocity changes. Mathematical expressions like mA, which keep on recurring, are usefully denoted by some name; and the conferring of a name unfortunately always tends to reify or hypostatize that which is thus denoted. But the whole tendency of modern experimental as well as mathematical physics is to eliminate the metaphysical notion of matter as an ultimate substance, and to find the element of permanence—without which there would be no science—in the mathematical relations. Thus Helmholtz, who in his youth thought that "the final aim of physical science is to find the ultimate unchangeable causes of the processes in nature," became satisfied later that the principle of causality meant nothing more than that natural phenomena happen according to law.[36]

We must then not only admit with Hume that conscious analysis does not show any single event to necessitate any other event, but modern physics suggests that the laws of nature which do correlate these events are themselves contingent, in the sense that they are known to be true only within the limits of observation, and may perhaps not prevail outside of the infinitesimal portion of the universe whose surface we have scratched. We can not be sure that these laws held true in the distant past any more than we can be certain that they now hold in the more distant parts of space not available to our instruments.[37]

It is not a valid objection to this view that it does violence to the

[36] *Wissenschaftliche Abhandlungen* (1882), p. 68.

[37] For a vigorous refutation of the easy assumption that the known laws of physics must hold for the whole universe see Chwolson in *Scientia,* 1910.

universal practical certainty as to the existence of a permanent and uniform nature of things. Certainty is a psychologic affair, and people are notoriously most certain on complicated questions of politics and religion, on which they have the least knowledge. But it is a valid argument against empiricist interpretation of it that it fails to account for the fundamental assumption underlying all scientific procedure: namely, that the logically necessary relations which hold between mathematical expressions hold of natural phenomena themselves. No physicist for a moment doubts that all the unforeseen logical consequences of a true physical hypothesis must necessarily hold for the physical universe in which that hypothesis is true, and that, if any of these consequences turn out to be false, it must be due to the falsity of our original assumption and not to the fact that nature fails to behave in accordance with the rules of mathematical deduction or computation. So long, therefore, as the laws of logic and mathematics are applicable to the physical universe, necessity of a certain kind, namely, the necessity which connects ground and consequent, must be predicated of it. It would not be difficult to show that this is precisely the necessity which common sense and physical science actually attribute to the causal relation. A stone thrown up *must* fall down after its upward velocity is spent and it has thus become a free body, if we assume, as we do, the law of gravity. If carbon combines with oxygen and thus burns, any substance like paper, made of wood pulp, *must* burn. The consequences in both cases are necessary and physically explained, though the major premises are contingent. If the law of gravitation or that carbon combines with oxygen could themselves be deduced from another law—for example, some law of electro-magnetism—the realm of physical explanation would be widened and greater unity be introduced. But the logical character of physical explanation would remain unaltered. Actually, the search for physical causes or explanations is, thus, a hunt for appropriate major premises or middle terms. The principle of causality (as distinct from particular causal laws) is thus simply the general maxim that physical phenomena are connected according to invariant laws. While this maxim is properly a postulate or resolution of the scientific understanding to look for such connections, it can be maintained only because the world of physics is full of universal elements or relations which repeat themselves indefinitely.[38]

The significance of this obvious truth, that logical or hypothetical necessity holds of nature, has been obscured by a number of

[38] Poincaré, "*Les equations expressent des rapports et si les equations restent vraies, c'est que ces rapports conservent leur realité.*" *Congrès des Phys.*, 1900, *Comptes Rendus*, I., p. 15.

powerful dogmas in modern philosophy which are certainly not the outgrowth of reflection on the nature of modern physics. These dogmas are: (1) that logical and mathematical relations and abstractions generally exist in the mind only, while physical phenomena exist in the external world; (2) that strict or deductive reasoning is a series of tautologies which can not extend our knowledge; (3) that science deals only with the actual existing world, and (4) the monistic or organic view of truth which fails to note that approximation or partial truths are still truths.

One would have to be devoid of a sense of humor to attempt in the limited time available for this paper, a complete refutation of all these dogmas. But I may suggest as a possible philosophic venture to note how dubious or in need of radical revision these dogmas appear, when we deal seriously with the fact that after all nature does behave in conformity with logical and mathematical principles. Consider, for instance, the following statements of Mach: "No one will fancy that vibrations in themselves have anything to do with circular functions or the motions of falling bodies with squares." "These mental expedients have nothing to do with the phenomenon itself." Here clearly Mach, the monistic sensationalist or empiricist, is at one with the metaphysical dualism of Descartes and his dogma that universals and principles are in the mind only, while the physical world of extension lies outside of it.[39] But this fails to explain why phenomena seem to occur as if the law of gravitation with its inverse squares were true, or why the properties of circular functions have proved most potent instruments for the discovery of important facts in almost all branches of physics. Doubtless, equations are not vibrating strings; but is it not straining the dualistic dogma to assert that they have nothing to do with each other? Do not let us be misled by the term expedient or invention. A map or chart is an expedient or invention. Yet if it fairly represents its object, is it not because certain relations between its parts are precisely those between the corresponding parts of the object represented? Mach admits that it is easier to deal with natural phenomena when the relations between the quantities investigated be similar to certain relations between familiar mathematical functions. But what does that similarity here mean, if not identity of mathematical relations? No one denies the suggestive value of popular analogies, such as that which speaks of certain social phenomena as periodic, rhythmic, or typifying the swing of the pendulum. Such analogies are dangerous, because popular language does not indicate clearly where the differences begin and where identity of relations ceases.

[39] Mach, *Mechanics*, pp. 492–494. Descartes, *Principia*, I., Art. 23.

When, however, different processes are expressed in analogous equations, the extent of the identical elements is unmistakably indicated, and deductions made from these equations are applicable to all the possible regions in which exist relations such as expressed in the equations before us.

Grant that the law of identity is not a mere tautology or an assertion how we do, as a matter of fact, think, but a significant assertion that there are elements which remain identical; grant, for example, that the relation represented by the ratio 2:1 may hold between all sorts of different entities, and most of the artificial problems of the classical epistemology disappear. At any rate, it becomes rather easy to explain the seeming paradox that physical laws may be true and yet physical phenomena show departure from them. For physical entities may have invariant relations between their parts and yet, being complexes, not have their entire character expressed by such simple laws or mathematical formulas containing a small number of factors or operations. Consider, for instance, Newton's first law of motion: "A body not acted on by force would continue in a state of rest or uniform motion in a straight line forever." If we adopted the mode of argument prevailing among certain contemporary philosophers, we should say: This is a foolish statement. No one has ever seen a body not acted on by any force, much less verified its motion forever. Indeed, no such body does exist, if Newtonian laws of universal gravitation be true. Yet, though no single physical body acts as if the law of inertia were the only law, the law of inertia is still an indispensable part of the Newtonian mechanics—which, with all its limitations, is still one of the most accurate descriptions of nature that the human mind has produced. Again, consider Boyle's law that the volume of a gas varies inversely as the pressure. There is not a single gas that conforms to it exactly, but it is not, therefore, false. It is a true first approximation, rendered more adequate when we introduce additional factors, as was done by Gay-Lussac and later by Van der Waals. As our instruments of precision and control over nature increase, we attempt to eliminate more and more of the residual variations. This seems an endless task, but the physicist must always assume that phenomena depend not upon an infinite but upon a very limited number of factors.

Note that dependence upon a limited number of factors means independence of all others. The organic point of view, or Mill's notion that the total state of the universe at any one time is the cause of the total state of the universe at the next moment, ignores this element of independence which our physics is constantly asserting.

(For example, the friction in a gas is independent of its temperature, *etc.*) It seems to me also quite clear that a principle such as Mill states would be inconsistent with the principle of causality as actually employed and as explained by a physicist like Maxwell. As stated by the latter, it is the following: "The difference between one event and another does not depend on the mere difference of the times or places at which they occur but only on differences in the nature, configuration, and motion of the bodies concerned."[40] If we did not eliminate from our consideration the particular moments of time or points of space at which events occur, physics would remain impossible. To speak of an event at all, there must be kinds or classes of them. They must be capable of repetition. And these repeatable time intervals (seconds, years, *etc.*) and space intervals (yards, *etc.*) rather than instants and points enter into the causal relation of actual physics. But the principle of causality as thus formulated carries us further and determines our choice of space reference and time measurement, as well as the form of our mathematical equations. Suppose I measure the length of a certain rod and find that it varies irregularly. The maxim of causality means that such variation is not due merely to the difference in times and spaces at which the measurement was made, but to other factors, such as heat, pressure, *etc.* Similarly, if I notice that the tide rises higher on some days than on others, the principle of causality means that it is not the time at which it occurs but certain factors such as winds, nearness of moon, *etc.*, which must be taken into account. No single law of physics would have meaning if everything depended upon everything else. If the freezing of water depended upon an infinite number of factors, there would be no sense in saying it depends upon temperature and pressure or that one of the latter can be varied while the other is constant. We can speak of water at all only because certain qualities or groups of qualities maintain their identity and keep on repeating themselves while other things change. The particular gunpowder whose explosion fires a particular shell will never again do so, but the elements of mass, units of force, velocity, *etc.*, will repeat themselves indefinitely.

Without the assumption of the existence of identical elements, all common sense, as well as scientific assertion, becomes not only false but meaningless. On the other hand, unless physical nature behaves according to the laws of diversity (excluded middle and contradiction) not a single mathematical principle could be applied to it, and it might, as far as physics is concerned, be one big blooming confusion. In all physical operations where addition is applicable, we

40 Maxwell, *Matter and Motion*, p. 31.

see operations which are essentially independent of each other. It is not necessary, for purposes of physics, to believe that nothing ever can happen except in accordance with the actually known laws of physics. It is not necessary to believe that the world exists solely for the satisfaction of our scientific ideals. But it is certainly most reasonable to suppose that the relations expressed by physical laws are actual constituents of the world, and that large domains of the latter are as described by our physical science.

To sum up: mechanism has failed as a final and complete account of physics. An adequate analysis of its progress bears out the contention that not ὑλή, formless matter or blind sensation, but mathematical and logical relations form the intelligible substance of things. But that the world contains more than this intelligible substance, our emotions and actions amply testify.

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REVIEWS AND ABSTRACTS OF LITERATURE

Elements of Constructive Philosophy. J. S. MACKENZIE. New York: The Macmillan Company. 1917. Pp. 487.

Let it be recognized at the outset that the present reviewer will probably do his book less than justice; it moves in the atmosphere of tradition, but the author is at home there with comfort and urbanity. When an interior is so agreeably furnished with excellent copies of the genius of the Greeks, of the Middle Ages, of the Renaissance, and of early Victorian inspiration, one easily forgets to go to the window and see how very different it looks outside. If to study philosophy is to go into a retreat, where one may wonder, in seclusion from the world, why facts are as they are and not otherwise, Dr. Mackenzie's book is an excellent companion to philosophic solitude. You will learn of much that many people have said about a great many things. I am not sure that you will learn what they were talking about at the time, nor why they talked about it, nor whether if they lived to-day they would go on talking about it. Our author is the accomplished host who knows how to conduct the conversation of his guests, giving to each his perfect opportunity, and refraining with considerate gentleness from speaking the deciding word.

We are informed in the preface that the present work was undertaken more than a quarter of a century ago, and that the author has had it pretty constantly in mind ever since. That accounts perhaps